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Application of Maximum Likelihood to the Gas Material Balance

Contrast to COGEH Guideline

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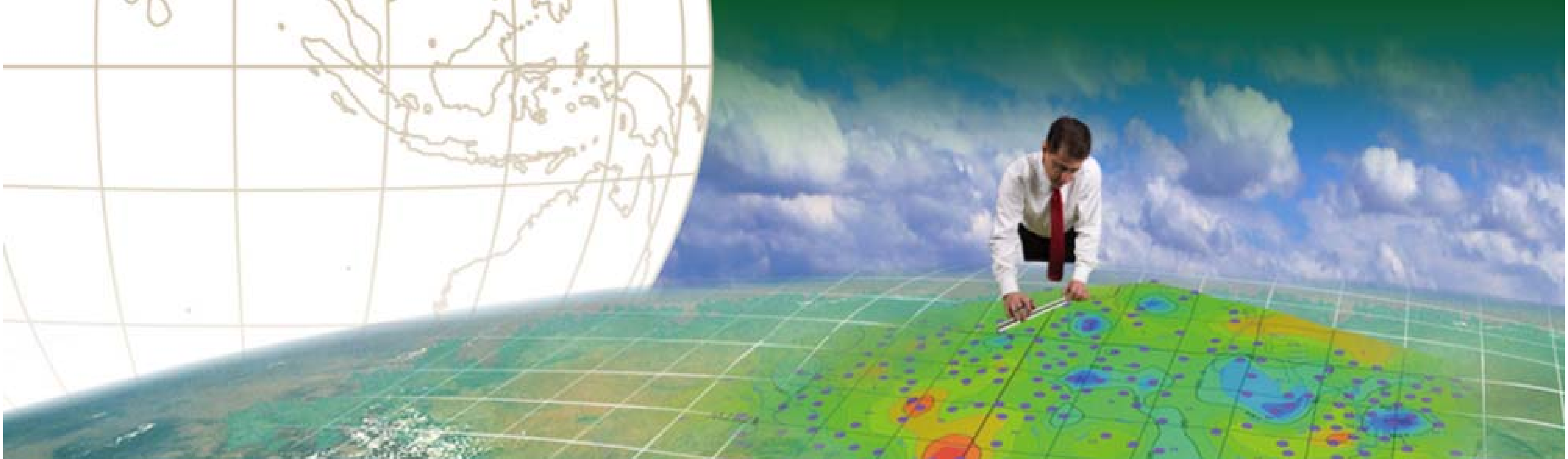


COGEH (paraphrased)

- P_{90} … at least a 90% probability that the quantities … will equal or exceed …
- P_{50} … at least a 50% probability …
- P_{10} … at least a 10% probability …

COGEH (direct extraction)

"A quantitative measure of the certainty levels pertaining to estimates prepared for the various reserves categories is desirable to provide a clearer understanding of the associated risks and uncertainties. However, the majority of reserves estimates will be prepared using deterministic methods that do not provide a mathematically derived quantitative measure of probability. In principle, there should be no difference between estimates prepared using probabilistic or deterministic methods."

COGEH Volume 2, page 3-8

COGEH (the problem)

*" … However, the **majority of reserves estimates will be prepared using deterministic methods that do not provide a mathematically derived quantitative measure of probability**."*

- ***This statement is only partly true. Statistical analysis of data fitting a deterministic model can quantify probability.***

COGEH (the solution)

- ***Make estimates based on the method of maximum likelihood.***

- Maximum likelihood …

  sounds exotic,

  looks hard,

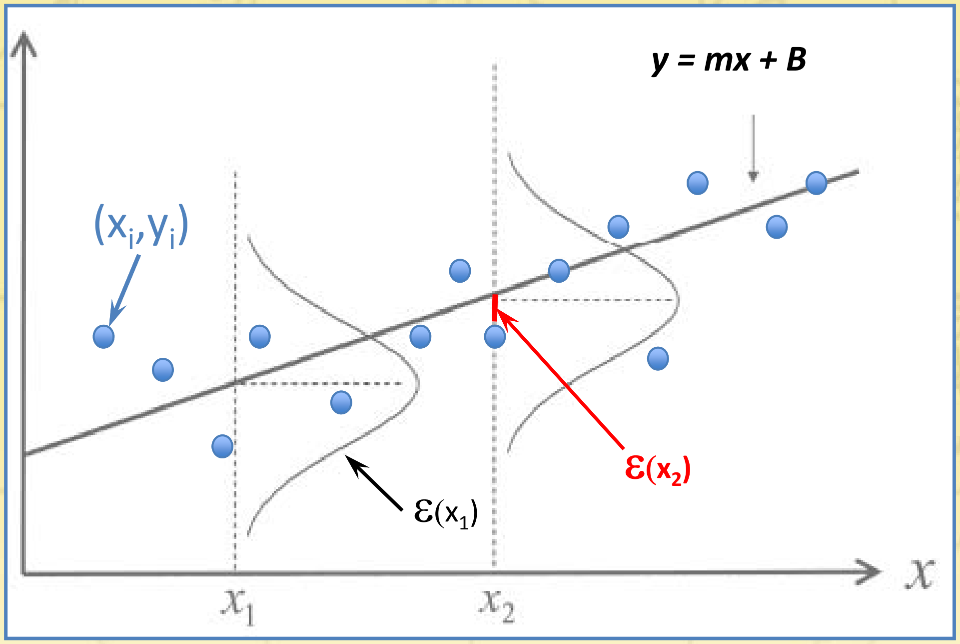
  yet, is quite easy.

Maximum Likelihood?

- Maximum likelihood estimation (MLE) is a strategy for obtaining statistically efficient parameter estimates.

- In layman's terms MLE provides a method which maximizes the likelihood (probability) that measured data fit a model, say for example the equation of a straight line.

Maximum Likelihood?



Model ……… y = mx + B

***Parameters*:**

- **m** = slope of line
- **B** = line's y-intercept

- $\varepsilon(x_i)$ = error at $x = x_i$

  $= y_i - (mx_i + B)$

  $= N(0, \sigma_i^2)$

Statistical Likelihood (looks hard)

The ***Likelihood*** function is defined as:

$$L = \prod_{i=1}^{n} \frac{1}{\sigma_i \sqrt{2\pi}} e^{-\frac{\varepsilon_i^{\,2}}{2\sigma_i^{\,2}}}$$

It is much more convenient for us to use ln(*L*)

$$\ln(L) = \sum_{i=1}^{n} \left[\ln\left(\frac{1}{\sigma_i \sqrt{2\pi}} \right) - \frac{\varepsilon_i^{\,2}}{2\sigma_i^{\,2}} \right]$$

Statistical Likelihood (looks hard)

ln(*L*) is more convenient to use because

- It is easily differentiated with respect to model parameters
- Maxima in ln(*L*) and *L* are at the same places

Gas Material Balance (model)

The conventional volumetric material balance equation for a single-phase gas reservoir is:

$$(P/Z) = (P/Z)_o\left(1 - \frac{G_p}{G}\right) = (P/Z)_o - \frac{(P/Z)_o}{G}G_p$$

Which has the form y = B – mx

Where:

$x_i = Gp_i$, $y_i = (P/Z)_i$

$B = (P/Z)_o$ and $G = B/m = (P/Z)_o/m$

MLE (only looks hard)

The intrinsic error at x_i for the material balance is:

$$\varepsilon_{(x_i)} = y_i - (B - mx_i)$$

$\ln(L)$ then becomes:

$$\ln(L) = \sum_{i=1}^{n} \left[\ln\left(\frac{1}{\sigma_i \sqrt{2\pi}} \right) - \frac{(y_i - B + mx_i)^2}{2\sigma_i^{\,2}} \right]$$

Basic Calculus (refresher)

For $Y = f(X)$

If a maxima exists it is located where $dY/dX = 0$

and

d^2Y/dX^2 must < 0

MLE (starting to look easy)

The maximum likelihood estimates for B and m occur where the following conditions are met:

$$\frac{\partial \ln(L)}{\partial m} = 0 = \sum_{i=1}^{n} \left[\frac{-2mx_i^{\,2} - 2x_i y_i + 2Bx_i}{2\sigma_i^{\,2}} \right]$$

$$\frac{\partial \ln(L)}{\partial B} = 0 = \sum_{i=1}^{n} \left[\frac{-2B + 2y_i + 2mx_i}{2\sigma_i^{\,2}} \right]$$

… since the respective 2nd derivatives are < 0, these are maximum likelihood estimates!

MLE for P/Z Mat. Balance (easy!)

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We have 2 equations with 2 unknowns (B & m) …

Solving for m, B and most importantly G …

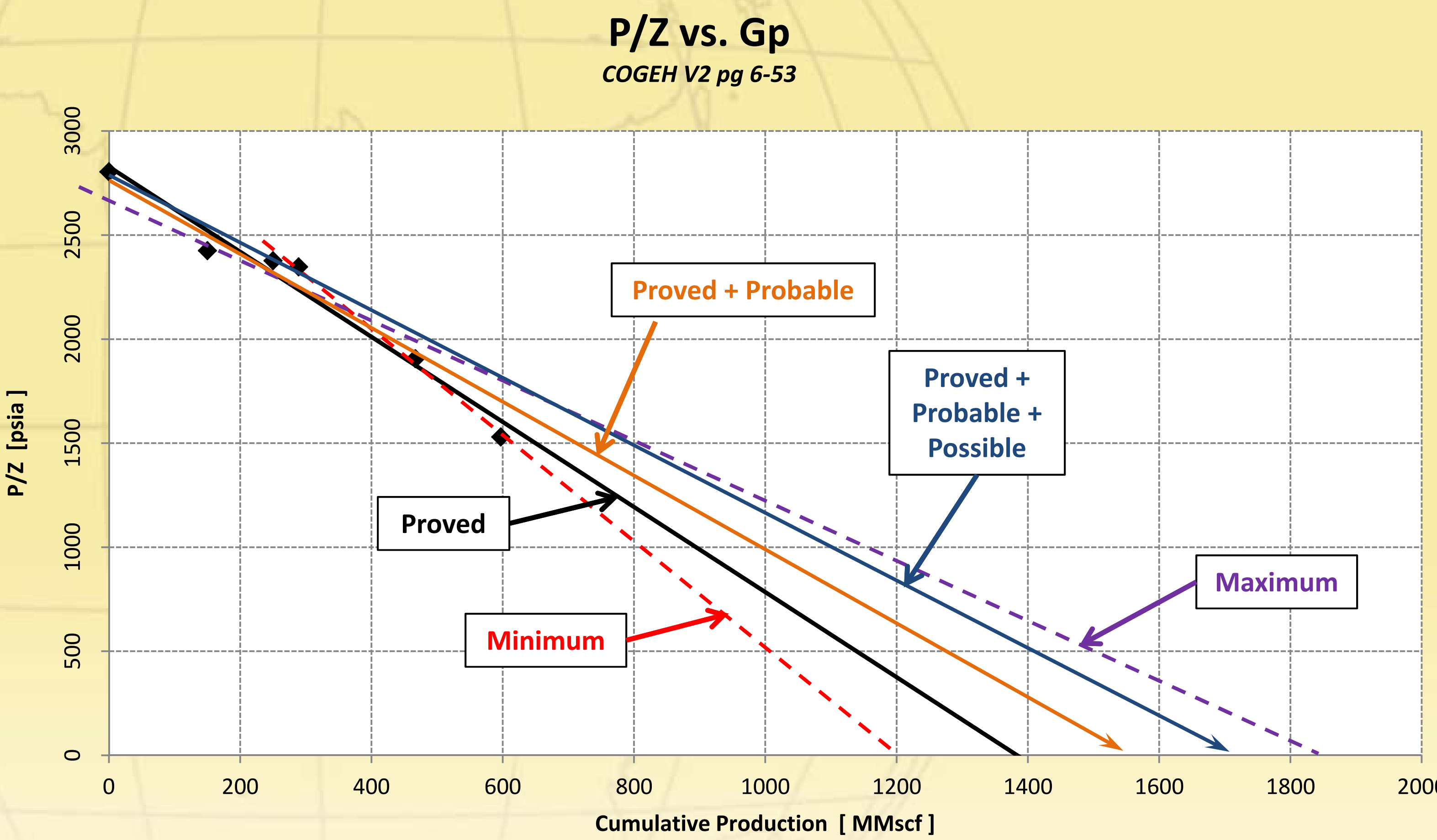
$$m = \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i - n\sum_{i=1}^{n} x_i y_i}{n\sum_{i=1}^{n} x_i^{\,2} - \left[\sum_{i=1}^{n} x_i\right]^2}$$

$$B = \left(\frac{P}{Z}\right)_o = \frac{\sum_{i=1}^{n} y_i + m\sum_{i=1}^{n} x_i}{n}$$

$$G = \frac{B}{m} = \frac{(P/Z)_o}{m}$$

Where: $x_i = Gp_i$, $y_i = (P/Z)_i$

COGEH Mat. Balance (arbitrary)



MLE Mat. Balance (statistical)

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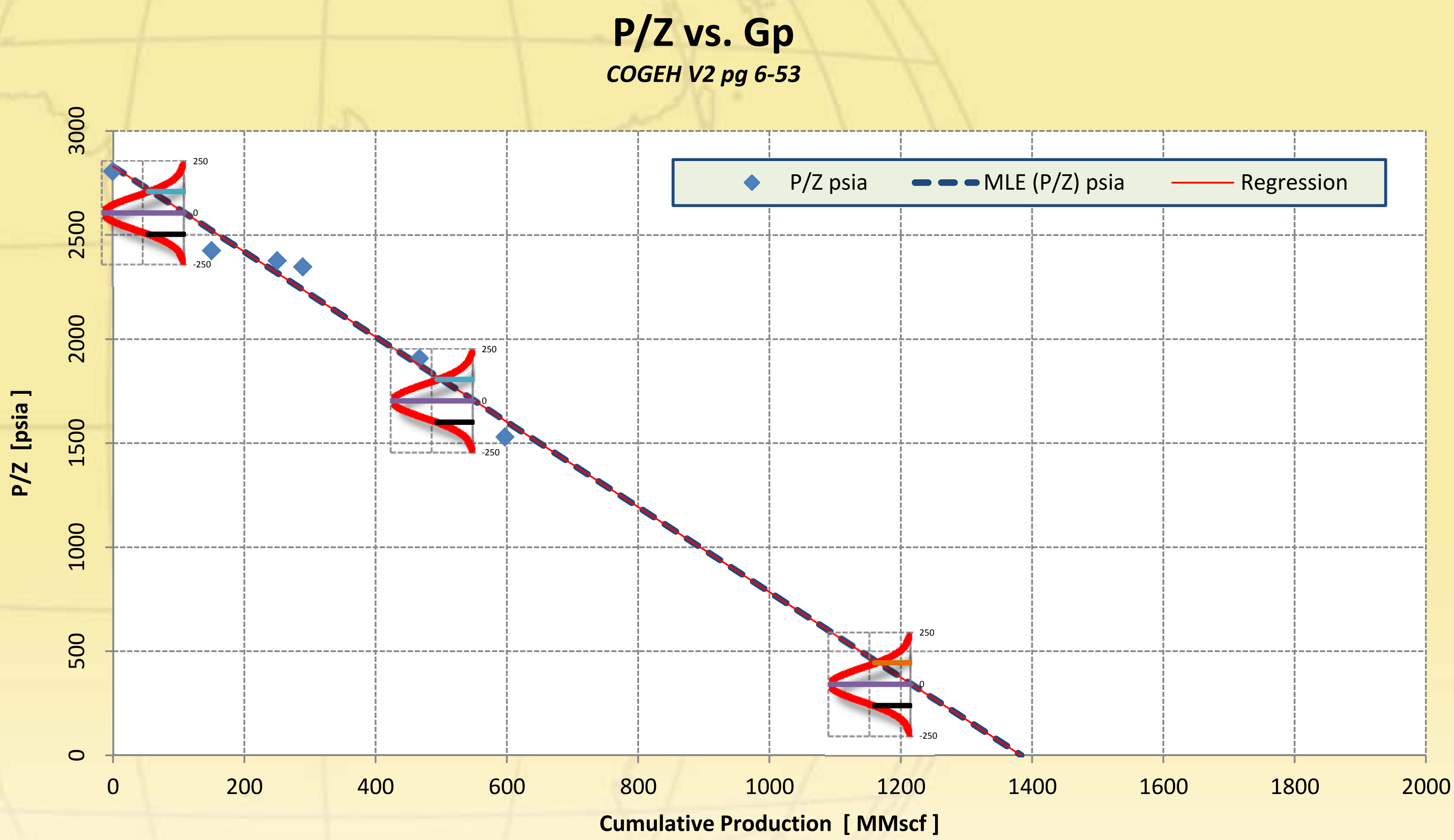

MLE Mat. Balance (statistical)

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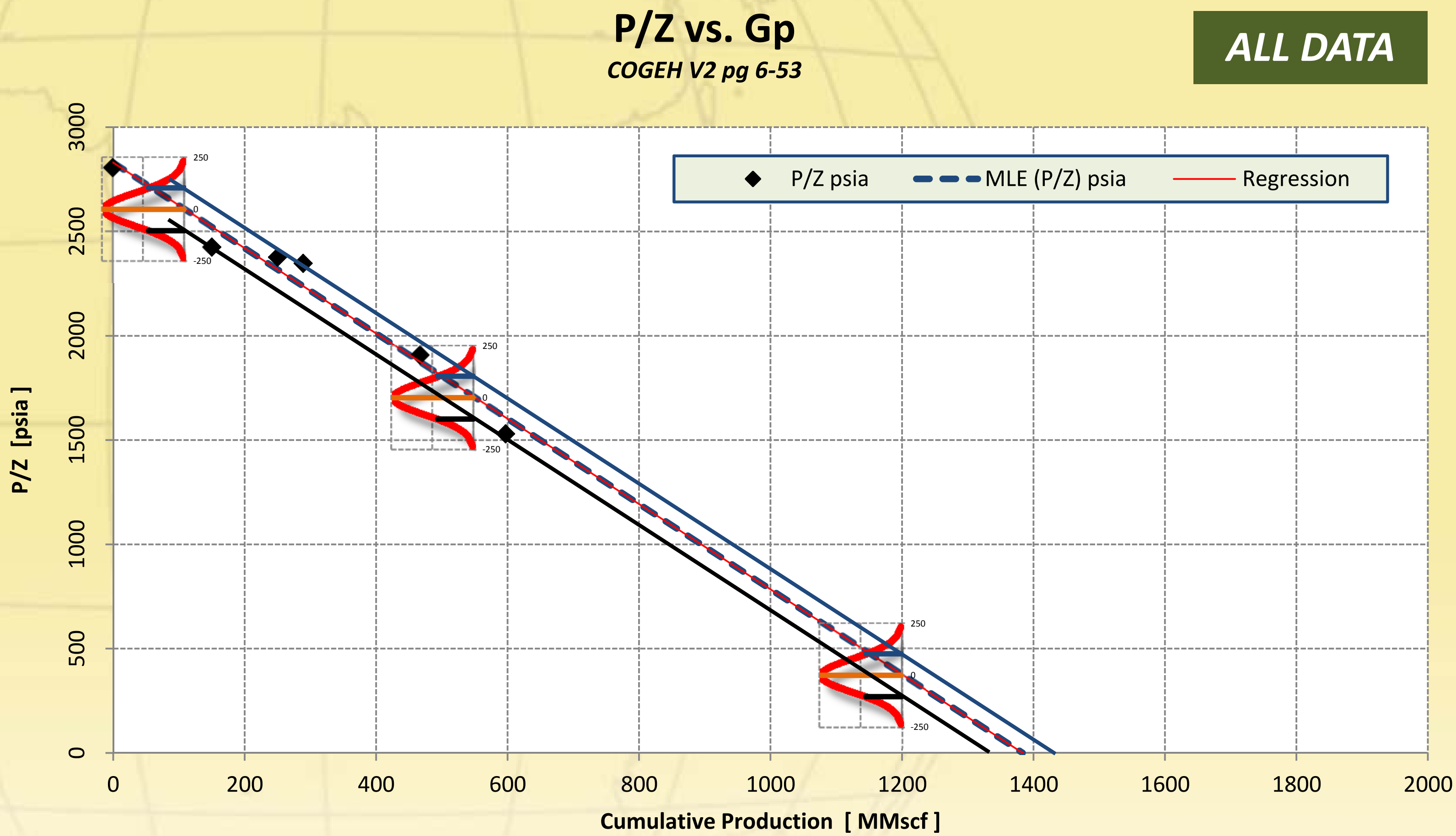

MLE Mat. Balance (statistical)

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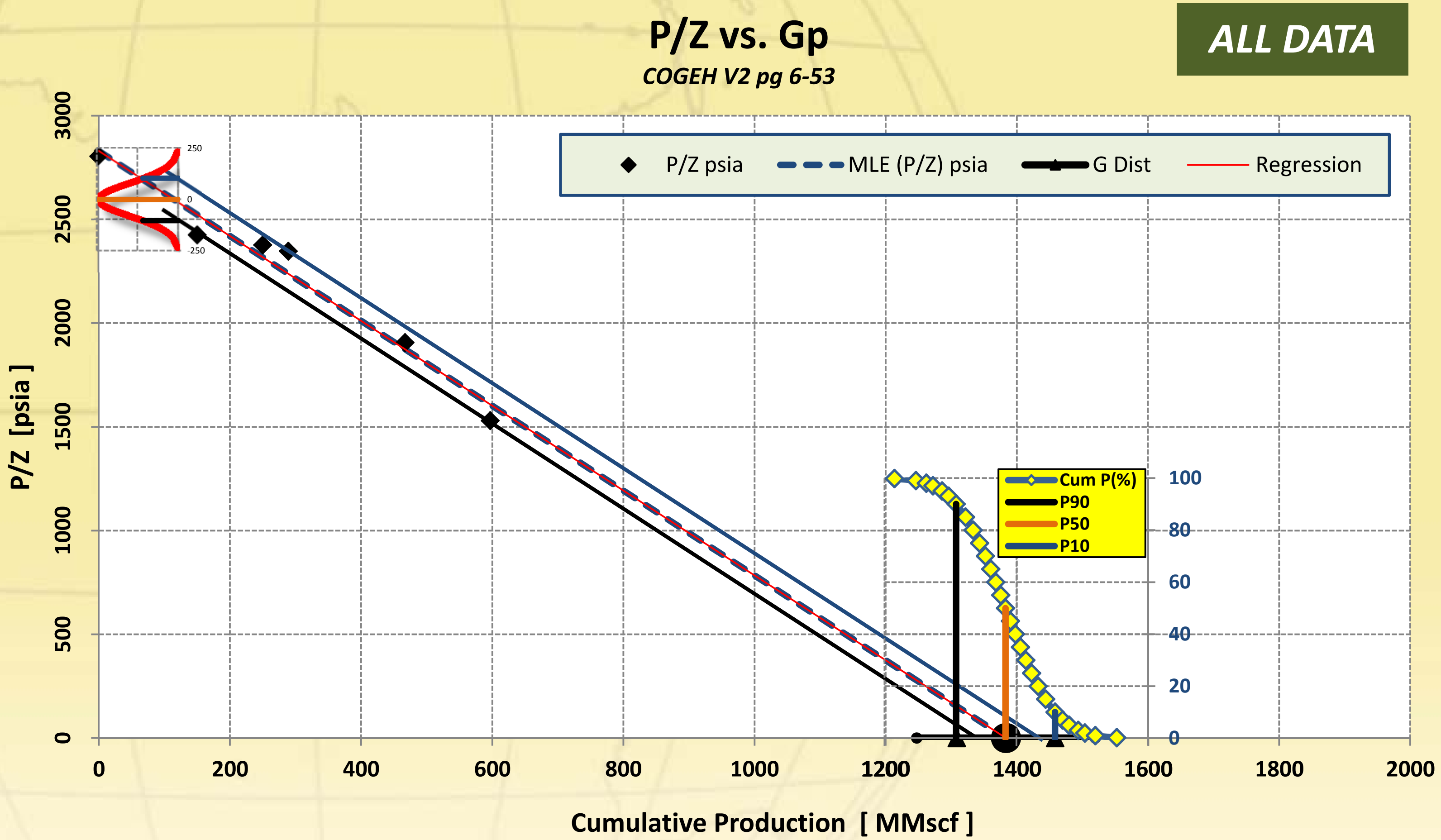
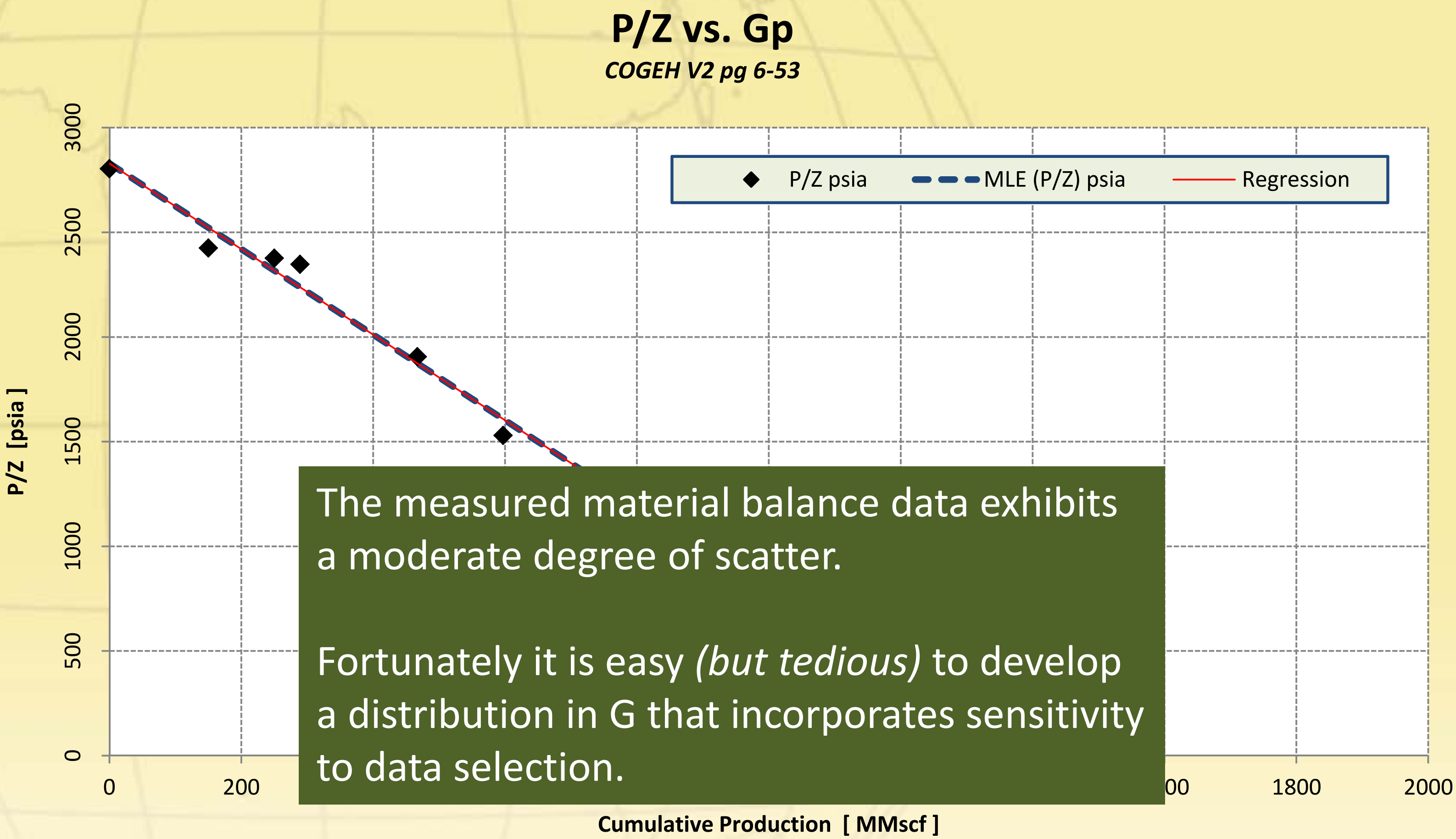
ALL DATA



MLE Mat. Balance (data sensitivity)



The measured material balance data exhibits a moderate degree of scatter.

Fortunately it is easy *(but tedious)* to develop a distribution in G that incorporates sensitivity to data selection.

MLE Mat. Balance (data sensitivity)

"n" normal distributions representing estimates of G can be derived by applying MLE to data subsets.

The overall distribution of G by simple averaging is:

$$G(\mu_G, \sigma_G{}^2) = \frac{1}{n}\sum_{i=1}^{n} G_i(\mu_i, \sigma_i{}^2)$$

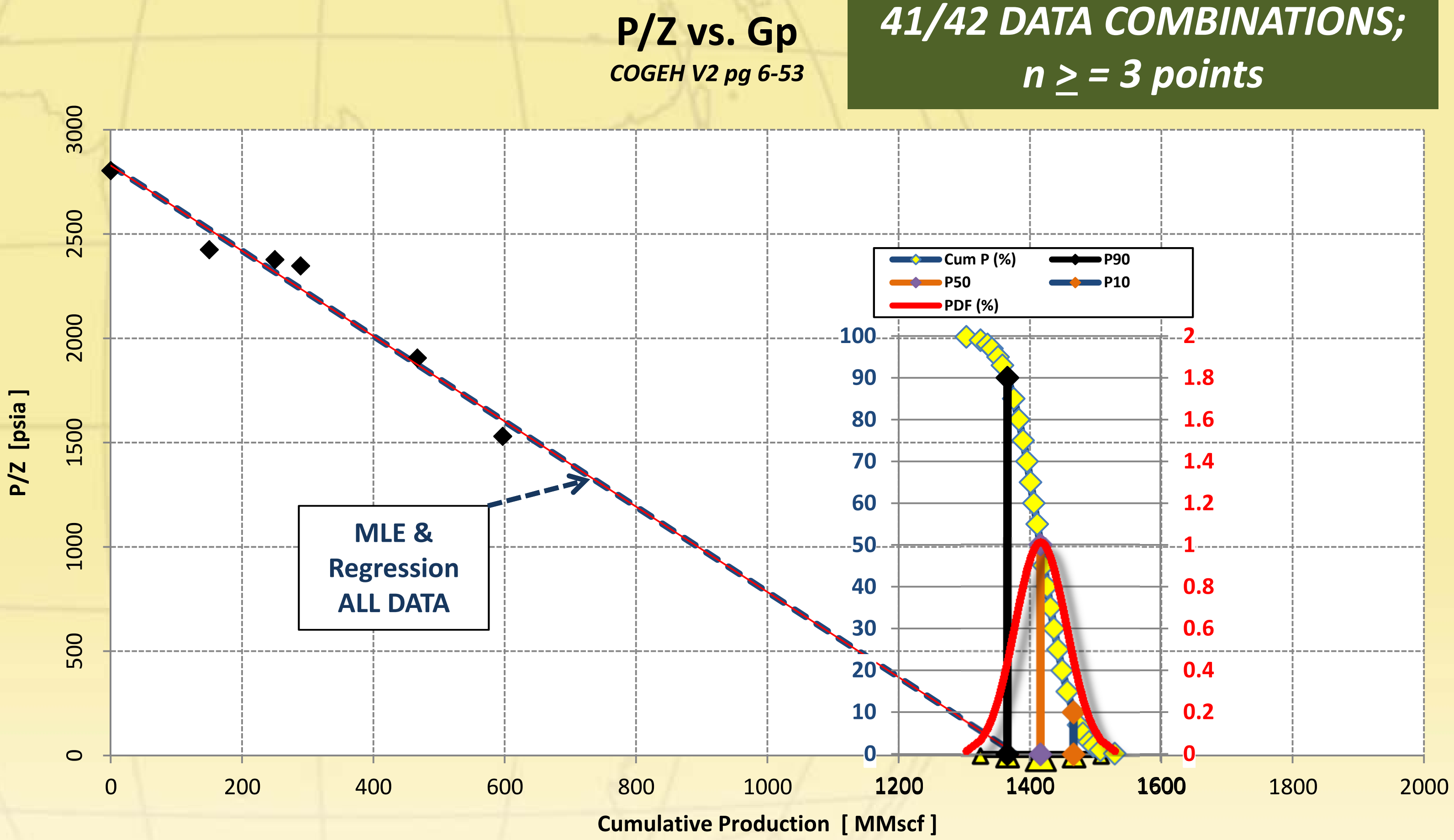
$$\mu_G = \frac{1}{n}\sum_{i=1}^{n} \mu_i$$

$$\sigma_G{}^2 = \frac{1}{n}\sum_{i=1}^{n} \sigma_i{}^2$$

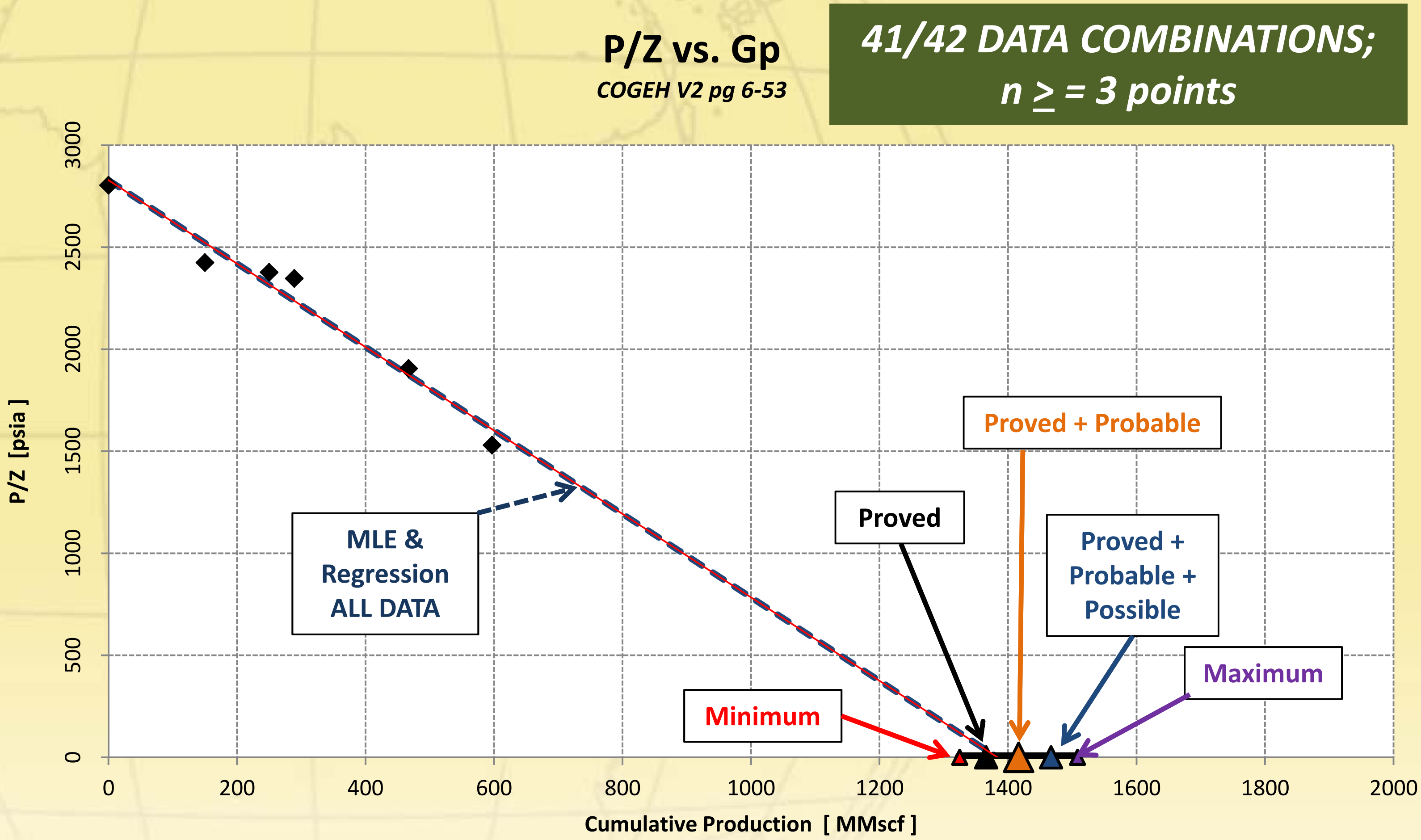
NOTE: **A weighted average formula for G, if desired, has an analogous form.**

MLE Mat. Balance (data sensitivity)

41/42 DATA COMBINATIONS;
n > = 3 points



MLE Mat. Balance Results



MLE vs. COGEH

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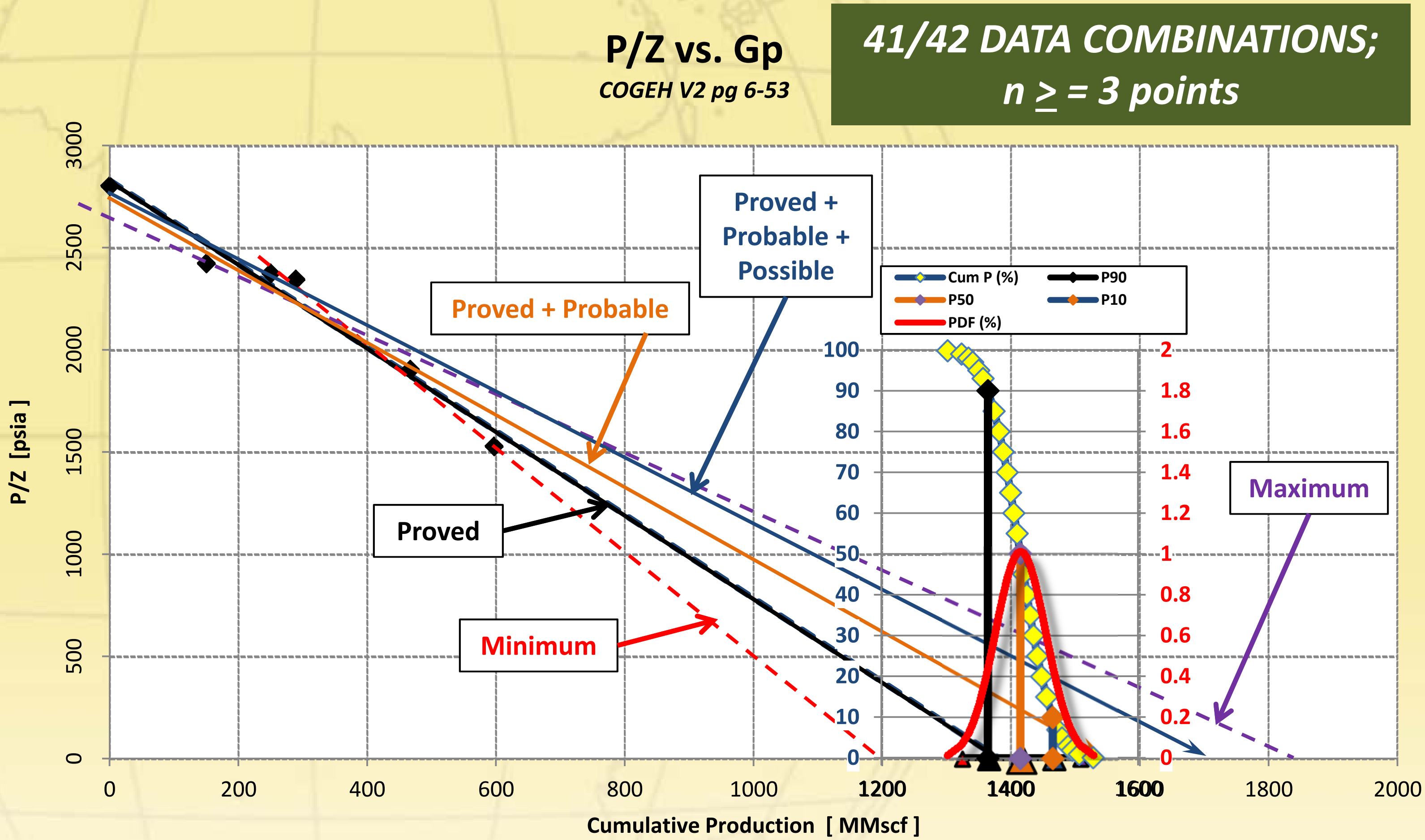

MLE vs. COGEH

	G (MMscf)		Δ
	COGEH	MLE*	(MLE-COGEH)
Minimum	1200	1181	-1.62%
P_{99}		1325	
Pv (P_{90})	1384	1366	-1.26%
Pv+Pb (P_{50})	1550	1417	-8.61%
Pv+Pb+Ps (P_{10})	1710	1467	-14.21%
P_{01}		1508	
Maximum	1840	1760	-4.36%

* 41/42 DATA COMBINATIONS; n $\geq$ 3 points

Conclusions

- The results of deterministic analyses can be assigned quantitative measures of probability.

- In this simple material balance example, the differences between COGEH P_{90}, P_{50} and P_{10} values of G and those estimated using probabilistic analysis of the data are large enough to be of concern. The differences are more pronounced at the P_{50} and P_{10} levels.